equal to 1, 2, ..., $\frac{n!}{(n-k)!k!}$.

The proof of this relation follows easily from (A), which is true for each set of k integers selected from the entire set of n integers. There will evidently be, in all, $\frac{n!}{(n-k)!k!}$ such relations. Multiplying together these relations, member by member, and noticing that in the middle member each of the n integers will occur $\frac{(n-1)!}{(n-k)!(k-1)!}$ times, the desired result is obtained.

3. If in (B) we set $k=2$, we obtain

$$(a_1 a_2 \ldots a_n)^{n-1} = \prod {}_2D_i \prod {}_2L_i, \quad i=1, 2, \ldots, \tfrac{1}{2}n(n-1),$$

which is probably the most general equality relation existing between integers, and greatest common divisors and least common multiples.

If, finally, $n=2$, we arrive at the well-known relation

$$a_1 a_2 = DL.$$

A POINT'S VISIT TO THE LINEAR CONTINUUM.

By H. W. REDDICK, Columbia University.

Once upon a time a point, XYZ, in three-dimensional space, decided to visit the set of points on a certain straight line. He had a sense of his superiority over his less fortunate fellow-beings who were compelled to lead a one-dimensional existence, but his motiqe for visiting them might be called philanthropic for he had a real desire to find out the relations existing between them and to enlighten them, if possible, concerning a higher existence.

As XYZ approached the line he was surprised to find that its residents were not living happily together; and he soon learned that they were capable of the same petty dissensions and jealousies which he had supposed were possible only in the enlightened set to which he belonged. He found that none of them was on speaking terms with his next-door neighbor — in fact, that he did not know who his next-door neighbor was. The society was divided up into political parties, religious denominations, and exclusiqe cliques, which spent most of their time quarrelling with one another, and did not seem to realize that, taken all together, they formed one Grand Continuum.

The society was divided into two great political sets, the Rationalists and the Irrationalists. The members of the Rational Set believed that the tariff on any commodity should be expressible in a finite number of figures. Their opponents could see no reason for such a belief. The chief argument of the Rationalists against the Irrationalists was that they were irrational. It was impossible, however, to settle any question by voting because there was always trouble in counting the Irrational vote.

There were two great religious denominations, the Transcendental and the Algebraic sects. The Transcendentalists looked upon those of the opposite faith with a feeling of pity, not unmingled with contempt, reproaching them for allowing themselves to be counted and for serving as roots of mere algebraic equations with rational coefficients. But the members of the Algebraic Set were firm in their pragmatic belief and denounced their opponents as being unpractical. Every inhabitant of the Continuum felt that he had the right of membership in one of these religious denominations but there was a great multitude who did not know to which set they should belong. However, there were two saints among the Transcendental Set whose right to be termed orthodox members of the faith had been established beyond doubt. These were e and π. Of these, e was the patron saint, who had been canonized by a human, Hermite, in 1873. e had become distinguished, however, long before this, by being appointed to act as base for the Napierian system of logarithms. But the members of the Transcendental Set never mentioned this distinction when eugolizing e in the presence of a member of the Algebraic Set, for the latter would promptly remind them that one of his number, 10, had achieved a greater glory by holding the office of base of the more practical Briggs system of logarithms. The other saint, π, had come up through great tribulation. Many were the mathematical sins that had been committed in his name! Owing to his pecaliar attribute of being the ratio of a circle to its diameter, he had often been misunderstood and cursed as a member of the Algebraic Set. But his vindication, glorious and complete, was brought about in 1882 by Lindemann. *XYZ* found that the two following questions were the ones most discussed by the Transcendentalists: Which of our number will be the next to be proved worthy of membership? Will a rule ever be found that can be applied to any individual to determine to which set he should belong?

Mr. *XYZ* then turned his attention to some of the more exclusive cliques that had recently been formed. There were several sets, regarded as snobbish by the others, who called themselves perfect, but they were all closed, so that *XYZ* failed to obtain much information about them. One of the most famous of these was Cantor's Typical Ternary Club, a very aristocratic organization whose members looked upon all non-members as the masses who were everywhere dense. The latter, however, referred sneeringly to the members of the organization as of content zero.

Mr. *XYZ* then dropped in at the Integral Club, where a number of in-

tegers had assembled. One of them informed XYZ that their members had been added together and had multiplied rapidly and in some instances one had been subtracted from another or a family had been divided, but that in no case had a number been produced which did not belong to the set. "However, we are a discreet set," he added with a smile. "We are careful not to let the root-extractor enter our midst." The members then began discussing their figures, the figures by which they were recognized. "I maintain that I have the most peculiar shape," said 6, "See! when I stand on my head I become 9." "That 's nothing, said 8, "When I lie down I become infinite." "But then you are no longer one of us," replied the others, "What shall we do with him for perpetrating such a joke? We 'll leave it to you, Mr. XYZ." "Well," said XYZ, "I think he should be killed. Let him that is greatest among you cast the first stone." They were all silent. Then Zero rolled along and tried to start an argument. "You do n't amount to anything anyhow," the others protested. "Just the same," retorted Zero, "I claim a distinction of which none of you can boast: a distinguished human, Mr. Russell, has dedicated a chapter to me."

At this stage Mr. XYZ, regarding his mission as hopeless, moved away into space, more firmly grounded in his conviction that the Continuum will never be well-ordered.

AN EXAMPLE OF THE USEFULNESS OF FOURIER'S THEOREM IN SEPARATING THE ROOTS OF EQUATIONS.

By L. R. MANLOVE, England.

Cases must occur in practice where the roots of an equation cannot be separated by any of the well-known easier methods and where Sturm's Theorem is inapplicable by reason of the labor which it involves. In such cases resort may be had to a combination of Fourier's Theorem with Lagrange's method of approximation, as shown below.

EXAMPLE.

Let us take the equation

$$x^{17}-35x^{15}+11x^{14}-1000x^{10}+2500x^{5}-151x^{3}+1=0.$$

A first application of Fourier's Theorem shows that there are:

(a) *Two positive roots:* One between 1 and 2; one between 5 and 6.

(b) *Three negative roots:* One between 0 and -1; one between -1 and -2; one between -6 and -7.